KSME Journal, Vol. 9, No. 2, pp. 177~186, 1995

Robust Control of Feedback Linearizable Large-Scale Systems

Myung-Chul Han*

(Received July 22, 1994)

The design of decentralized controllers for a class of uncertain interconnected nonlinear systems is considered. The uncertainty considered here is time-varying and appears at each subsystem and interconnections. Two control techniques are explored. The first one, namely, the feedback linearization control, involves a known and autonomous nonlinear system. The second one, namely, the robust control, is especially suitable if any uncertainty and/or time-varying factors are involved in the nonlinear dynamics. These two controllers are combined to stabilize a class of large-scale nonlinear uncertain systems. Two decentralized robust controllers, non-adaptive and adaptive, are proposed and those results are proved.

Key Words: Decentralized Control, Robust Control, Feedback Linearization, Large-Scale Systems

1. Introduction

Modeling of large-scale systems is often described by a set of interconnected subsystems. The considered uncertainties are nonlinear (possibly fast) time-varying and are distributed into the inner portions of the subsystems and the interconnections. In practice, it may be difficult to acquire their real statistical properties *a priori*. Under these circumstances, it may be desirable to adopt a deterministic approach which is based on the possible bounds of the uncertainties.

Decentralized control is an effective way for the large-scale systems since each subsystem can be independently controlled. Important work on decentralized control of large-scale uncertain systems can be found in (Chen, 1988 ; Chen and Han, 1993 ; Han and Chen, 1991, 1992b ; Gavel and Siljak, 1989 ; Ikeda and Siljak, 1990 ; Ohta *et al.*, 1986 ; Park and Lee, 1993; Siljak, 1989) and their bibliographies. In (Chen, 1988), two classes of control schemes, namely, the *local* and the *global*, are proposed. The local control is based on the state information of only each subsystem, while the global control utilizes the extra feedback information from the states of neighboring subsystems. This work falls into the local category.

Feedback linearization provides a unified approach for the design of nonlinear system controllers. However, the robustness of the resulting controller can not be guaranteed. Several methods have been used together with feedback linearization to increase the robustness of the control. These include, for example, variable structure control (Sira-Ramirez, 1986), adaptive control (Sastry and Isidori, 1989 ; Taylor *et al.*, 1989) and Lyapunov based method (Spong and Sira-Ramirez, 1986 ; Calvet and Arkun, 1989). In this paper, the results of the Lyapunov based approach are extended to large-scale systems.

A diffeomorphic state transformation is first applied to each subsystem. The transformation together with a nonlinear control render the nominal system to be controllable-like. An extra control effort is then added to the nonlinear controller to ensure satisfactory results even if the uncertainties appear. Two control schemes, namely, non-adaptive and adaptive robust controls, are

* Department of Control and Mechanical Engineering, Institute of Mechanical Technology, Pusan National University, 30 Changjeon-dong, Kumjeong-ku, Pusan, 609-735, Korea

proposed. In the non-adaptive robust control design, all the bounds of uncertainties are given and the control gain parameters are related to the bounds. In the adaptive robust control design, this prerequisite of the bounds can be relaxed if each subsystem does not have uncertainty in the input matrix. However, an additional requirement on the uncertainty bounds, namely, cone-boundedness, should be satisfied. An adaptive algorithm is adopted to track the bounds of uncertainties of each subsystem and the control is based on this estimation. The related work can be found in (Chen, 1990 ; Corless and Leitmann, 1983).

The main contributions of this work can be divided into three parts. First, it is shown that feedback linearization is sucessfully combined with robust control to supply a systematic control design method for nonlinear large-scale uncertain systems. Second, decentralized robust controls are proposed, where the bounds of interconnections are explicitly taken into acconunt while the neighboring states are not required. Third, an adaptive algorithm is adopted to guarantee the desired properties in spite of insufficient information of uncertainty bounds.

2. Interconnected Systems

We consider a class of uncertain large-scale systems S which are composed of N interconnected subsystems S_i described by

$$\begin{aligned} S_i : \dot{x}_i(t) &= f_i(x_i(t)) + \Delta f_i(x_i(t),\ \sigma_i(t)) \\ &\quad + [g_i(x_i(t)) + \Delta g_i(x_i(t), \\ &\quad \sigma_i(t))]u_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} r_{ij}(x_i(t), \\ &\quad x_j(t),\ \sigma_i(t)) \\ x_i(0) &= x_{i0} \end{aligned} \tag{1}$$

for all $i \in \mathrm{N}$, $\mathrm{N} \equiv \{1,\ 2, \cdots, N\}$. Here $t \in \mathrm{R}$ is the time, $x_i(t) \in \mathrm{R}^{n_i}$ is the state, $u_i(t) \in \mathrm{R}$ is the control, and $\sigma_i(t) \in \mathrm{R}^{q_i}$ is the uncertainty. Both the internal uncertainty (i.e., Δf_i and Δg_i) and uncertainty in the interconnections (i.e., r_{ij}) are considered. It is assumed that the unkown funtion $\sigma_i(\cdot)$: $\mathrm{R} \rightarrow \sum_i$ is Lebesgue measurable where $\sum_i \subset \mathrm{R}^{q_i}$ is a compact set (known or unknown). Furthermore, the known functions $f_i(\cdot)$:$\mathrm{R}^{n_i} \rightarrow \mathrm{R}^{n_i}$ and $g_i(\cdot)$:$\mathrm{R}^{n_i} \rightarrow \mathrm{R}^{n_i}$ and the functions (known or unknown) $\Delta f_i(\cdot)$:$\mathrm{R}^{n_i} \times \sum_i \rightarrow \mathrm{R}^{n_i}$, $\Delta g_i(\cdot)$:$\mathrm{R}^{n_i} \times \sum_i \rightarrow \mathrm{R}^{n_i}$, and $r_{ij}(\cdot)$:$\mathrm{R}^{n_i} \times \mathrm{R}^{n_j} \times \sum_i \rightarrow \mathrm{R}^{n_i}$ are continuous.

The following assumptions are introduced.

Assumption 1: There exist continuous functions $h_i(\cdot)$: $\mathrm{R}^{n_i} \times \sum_i \rightarrow \mathrm{R}$, $l_i(\cdot)$: $\mathrm{R}^{n_i} \times \sum_i \rightarrow \mathrm{R}$, and $c_{ij}(\cdot)$: $\mathrm{R}^{n_j} \times \sum_i \rightarrow \mathrm{R}$ such that

$$\begin{aligned} \Delta f_i(x_i,\ \sigma_i) &= g_i(x_i)\, h_i(x_i,\ \sigma_i) \\ \Delta g_i(x_i,\ \sigma_i) &= g_i(x_i)\, l_i(x_i,\ \sigma_i) \\ r_{ij}(x_i,\ x_j,\ \sigma_i) &= g_i(x_i)\, c_{ij}(x_i,\ x_j,\ \sigma_i) \\ \rho_{li} &\equiv \min\ \sigma_i \in \textstyle\sum_i l_i(x_i,\ \sigma_i) \\ &> -1 \end{aligned} \tag{2}$$

for all $(x_i,\ x_j) \in R^{n_i} \times R^{n_j}$ and $\sigma_i \in \sum_i$.

With this assumption, The system equations (1) can be written as

$$\begin{aligned} S_i : \dot{x}_i &= f_i(x_i) + g_i(x_i)[h_i(x_i, \sigma_i) \\ &\quad + (1 + l_i(x_i,\ \sigma_i))\, u_i \\ &\quad + \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij}(x_i,\ x_j,\ \sigma_i)] \end{aligned} \tag{3}$$

Remark : Assumption 1 assures that all uncertain portions Δf_i, Δg_i, and r_{ij} in each subsystem are contained in the range space of the nominal input matrix g_i. This structural condition on the uncertainty is usually called the matching condition (e.g., Leitmann, 1891 ; Gavel and Siljak, 1989). Work on the relaxation of the matching condition can be found in, e.g., (Chen and Leitmann, 1987 ; Ikeda and Siljak, 1985).

For convenience, we define

$$\begin{aligned} x &\equiv [x_1^T,\ x_2^T, \cdots, x_N^T]^T \in \mathrm{R}^n,\quad n = \sum_{i=1}^{N} n_i \\ x_0 &\equiv [x_{10}^T,\ x_{20}^T, \cdots, x_{N0}^T]^T \in \mathrm{R}^n \\ u &\equiv [u_1,\ u_2, \cdots, u_N]^T \in \mathrm{R}^N \\ \sigma &\equiv [\sigma_1^T,\ \sigma_2^T, \cdots, \sigma_N^T]^T \in \mathrm{R}^q,\quad q = \sum_{i=1}^{N} q_i \\ f(x) &\equiv [f_1(x_1)^T,\ f_2(x_2)^T, \cdots, \\ &\quad f_N(x_N,\ \sigma_N)^T]^T \in \mathrm{R}^n \\ \Delta f(x, \sigma) &\equiv [\Delta f_1(x_1, \sigma_1)^T,\ \Delta f_2(x_2, \sigma_2)^T, \cdots, \\ &\quad \Delta f_N(x_N, \sigma_N)^T]^T \in R^n \\ G(x) &\equiv \mathrm{diag}\{g_1(x_1)^T,\ g_2(x_2)^T, \cdots, \\ &\quad g_N(x_N)\}^T \in \mathrm{R}^{n \times N} \\ \Delta G(x,\ \sigma) &\equiv \mathrm{diag}\{\Delta g_1(x_1,\ \sigma_1)^T, \\ &\quad \Delta g_2(x_2,\ \sigma_2)^T, \cdots, \\ &\quad \Delta g_N(x_N,\ \sigma_N)^T\}^T \in \mathrm{R}^{n \times N} \end{aligned}$$

$$R(x, \sigma) = \left[\left(\sum_{\substack{j=1\\j\neq 1}}^{N} r_{1j}(x_1, x_j, \sigma_1)\right)^T, \left(\sum_{\substack{j=1\\j\neq 2}}^{N} r_{2j}(x_2, x_j, \sigma_2)\right)^T, \cdots, \left(\sum_{\substack{j=1\\j\neq N}}^{N} r_{Nj}(x_N, x_j, \sigma_N)\right)^T\right]^T \in \mathbf{R}^n$$

The following definition describes the desired system behavior.

Definition 1: Given a control $u(t) = \bar{p}(x(t))$, the resulting closed-loop large-scale uncertain system

$$\begin{aligned} \dot{x} &= f(x) + \Delta f(x, \sigma) + [G(x) + \Delta G(x, \sigma)]\bar{p}(x) + R(x, \sigma) \\ x(t_0) &= x_0 \end{aligned} \tag{4}$$

is practically stable iff there exists an $r_0 > 0$ such that the following properties hold.

(i) Existence of solutions : The system (4) possesses a solution $x(\cdot) : [t_0, \infty) \to \mathbf{R}^n$.

(ii) Uniform boundedness : Given any $\underline{r} \in (0, \infty)$ and any solution $x(\cdot) : [t_0, \infty) \to \mathbf{R}^n$ of Eq. (4), there exists a $d(\underline{r}) < \infty$ such that $\|x_0\| \leq \underline{r}$ implies $\|x(t)\| \leq d(\underline{r})$ for all $t \in [t, \infty)$.

(iii) Uniform ultimate boundedness : Given any $\bar{r} > r_0$ and any $\underline{r} \in (0, \infty)$, there exists a finite time $T(\underline{r}, \bar{r})$ such that $\|x_0\| \leq \underline{r}$ implies $\|x(t)\| \leq \bar{r}$ for all $t \geq t_0 + T(\underline{r}, \bar{r})$.

(iv) Uniform stability : Given any $\bar{r} > r_0$, there exists a $\delta(\bar{r}) > 0$ such that $\|x_0\| \leq \delta(\bar{r})$ implies $\|x(t)\| \leq \bar{r}$ for all $t \geq t_0$.

Throughout, we adopt the Euclidean vector norm. Matrix norm is the corresponding induced one.

3. Input-State Feedback Linearization

The nominal part of the system (1), which is represented by

$$S_i : \dot{x}_i = f_i(x_i) + g_i(x_i) u_i \tag{5}$$

is assumed to be globally input-state feedback linearizable, that is, for every nominal subsystem, there exists a global diffeomorphism $T_i(\cdot) : \mathbf{R}^{n_i} \to \mathbf{R}^{n_i}$,

$$\begin{aligned} T_i &= [T_{i1}, T_{i2}, \cdots, T_{in_i}]^T \\ z_{i1} &= T_{i1}(x_i) \\ z_{i2} &= T_{i2}(x_i) = L_{f_i} T_{i1}(x_i) \\ &\cdots \\ z_{in_i} &= T_{in_i}(x_i) = L_{f_i}^{n_i-1} T_{i1}(x_i) \end{aligned} \tag{6}$$

such that following holds:

$$\begin{aligned} &L_{g_i} L_{f_i}^k T_{i1} = 0 \text{ for } k = 0, \cdots, n_i - 2 \\ &L_{g_i} L_{f_i}^{n_i-1} T_{i1} \neq 0 \end{aligned} \tag{7}$$

Here $L_{f_i} T_{i1}$ stands for the Lie derivative of T_{i1} with respect to f_i:

$$\begin{aligned} &L_{f_i}^0 T_{i1} = T_{i1} \\ &L_{f_i}^1 T_{i1} = \nabla T_{i1} f_i \\ &L_{f_i}^j T_{i1} = L_{f_i} L_{f_i}^{j-1} T_{i1}, \quad j > 1 \end{aligned} \tag{8}$$

Without losing generality, it is assumed that the transformation T_i maps origin to origin.

From Eqs. (6)~(8), the transformed state dynamics are governed by

$$\begin{aligned} \dot{z}_{i1} &= (\nabla T_{i1}) f_i + (\nabla T_{i1}) g_i \left[h_i + (1 + l_i) u_i + \sum_{\substack{j=1\\j\neq i}}^{N} c_{ij}\right] \\ &= L_{f_i} T_{i1} + (L_{g_i} T_{i1}) \left[h_i + (1 + l_i) u_i + \sum_{\substack{j=1\\j\neq i}}^{N} c_{ij}\right] \\ &= z_{i2} \\ \dot{z}_{i2} &= (\nabla L_{f_i} T_{i1}) f_i + (\nabla L_{f_i} T_{i1}) g_i \left[h_i + (1 + l_i) u_i + \sum_{\substack{j=1\\j\neq i}}^{N} c_{ij}\right] \\ &= L_{f_i}^2 T_{i1} + (L_{g_i} L_{f_i} T_{i1}) \left[h_i + (1 + l_i) u_i + \sum_{\substack{j=1\\j\neq i}}^{N} c_{ij}\right] \\ &= z_{i3} \\ &\cdots \\ \dot{z}_{i(n_i-1)} &= (\nabla L_{f_i}^{n_i-2} T_{i1}) f_i + (\nabla L_{f_i}^{n_i-2} T_{i1}) g_i \left[h_i + (1 + l_i) u_i + \sum_{\substack{j=1\\j\neq i}}^{N} c_{ij}\right] \\ &= L_{f_i}^{n_i-1} T_{i1} + (L_{g_i} L_{f_i}^{n_i-2} T_{i1}) \left[h_i + (1 + l_i) u_i + \sum_{\substack{j=1\\j\neq i}}^{N} c_{ij}\right] \\ &= z_{in_i} \\ \dot{z}_{in_i} &= (\nabla L_{f_i}^{n_i-1} T_{i1}) f_i + (\nabla L_{f_i}^{n_i-1} T_{i1}) g_i \left[h_i + (1 + l_i) u_i + \sum_{\substack{j=1\\j\neq i}}^{N} c_{ij}\right] \\ &= L_{f_i}^{n_i} T_{i1} + (L_{g_i} L_{f_i}^{n_i-1} T_{i1}) \Big[h_i \end{aligned}$$

$$+(1+l_i)u_i+\sum_{\substack{j=1\\j\neq i}}^{N}c_{ij}\Big] \quad (9)$$

Choose the control input as

$$u_i=\frac{1}{L_{gi}L_{fi}^{n_i-1}T_{i1}}(\nu_i-L_{fi}^{n_i}T_{i1}) \quad (10)$$

For convenience, we define

$$z\equiv[z_1^T,\ z_2^T,\cdots,z_N^T]^T\in\mathrm{R}^n$$
$$\bar{l}_i(z_i,\ \sigma_i)\equiv l_i(x_i,\ \sigma_i)$$
$$\bar{h}_i(z_i,\ \sigma_i)\equiv h_i(x_i,\ \sigma_i)$$
$$\bar{c}_{ij}(z_i,\ z_j,\ \sigma_i)\equiv c_{ij}(x_i,\ x_j,\ \sigma_i)$$

Equation (9) becomes

$$\dot{z}_i=A_iz_i+B_i[1+\bar{l}_i(z_i,\ \sigma_i)]\nu_i+B_i[\omega_{i1}(z_i,\ \sigma_i)+\omega_{i2}(z,\ \sigma_i)] \quad (11)$$

where the matrices A_i and B_i are given by

$$A_i=\begin{bmatrix}0^{(n_i-1)\times 1} & I^{(n_i-1)\times(n_i-1)}\\ 0^{1\times 1} & 0^{1\times(n_i-1)}\end{bmatrix}$$
$$B_i=[0^{1\times(n_i-1)}\quad I^{1\times 1}] \quad (12)$$

and $\omega_{i1}(z_i,\ \sigma_i)$ and $\omega_{i2}(z_i,\ z_j,\ \sigma_i)$ are defined as

$$\omega_{i1}(z_i,\ \sigma_i)\equiv L_{gi}L_{fi}^{n_i-1}T_{i1}\bar{h}_i(z_i,\ \sigma_i)-\bar{l}_i(z_i,\ \sigma_i)L_{fi}^{n_i}T_{i1} \quad (13)$$

$$\omega_{i2}(z,\ \sigma_i)\equiv L_{gi}L_{fi}^{n_i-1}T_{i1}\sum_{\substack{j=1\\j\neq i}}^{N}\bar{c}_{ij}(z_i,\ z_j,\ \sigma_i) \quad (14)$$

Assumption 2 : For each i, $j\in$N, there exist known and non-negative scalar constants a_{ij} and b_i such that

$$\max_{\sigma_i\in\Sigma_i}|\ \omega_{i2}(z,\ \sigma_i)\ |\le\sum_{j=1}^{N}a_{ij}\|z_j\|+b_i \quad (15)$$

for all $z\in\mathrm{R}^n$.

4. Non-Adaptive Robust Control Design

The following decentralized non-adaptive robust control is proposed

$$\nu_i(z_i)=K_iz_i-\gamma_iB_i^TP_iz_i-p_i(z_i) \quad (16)$$

where K_i is chosen such that $A_i+B_iK_i$ is asymptotically stable, and matrix $P_i>0$ is the solution of the following Lyapunov equation

$$P_i(A_i+B_iK_i)+(A_i+B_iK_i)^TP_i=-Q_i,\quad Q_i>0 \quad (17)$$

The scalar γ_i is positive and is specified as following : First, we choose a constant δ_i such that

$$\delta_i>\frac{\sum_{j=1}^{N}a_{ji}}{\lambda_{\min}(Q_i)} \quad (18)$$

Second, we choose γ_i in the control (16) such that

$$\gamma_i>\frac{\sum_{j=1}^{N}\delta_ja_{ij}}{2(1+\rho_{li})} \quad (19)$$

The reason why these values are chosen will be stated later in stability analysis.

The saturation type control term $p_i(z_i)$ is chosen as following:

$$p_i(z_i)=\begin{cases}\dfrac{\mu_i(z_i)}{|\ \mu_i(z_i)\ |}\rho_i(z_i) & \text{if } |\ \mu_i(z_i)\ |>\varepsilon_i\\ \dfrac{\mu_i(z_i)}{\varepsilon_i}\rho_i(z_i) & \text{if } |\ \mu_i(z_i)\ |\le\varepsilon_i\end{cases} \quad (20)$$

$$\rho_i(z_i)\ge(1+\rho_{li})^{-1}|\ \omega_{i1}(z_i,\ \sigma_i)+\bar{l}_i(z_i,\ \sigma_i)K_iz_i\ | \quad (21)$$

$$\mu_i(z_i)=B_i^TP_iz_i\rho_i(z_i) \quad (22)$$

where ε_i is a positive constant.

Remark : The nominal control K_iz_i and any one of last two terms in Eq. (16) may render the subsystem (11) practically stable if there is no interconnections (Corless and Leitmann, 1981 ; Barmish *et al*., 1983).

In order to investigate the controlled system performance, we proceed with the following analysis. First, take a C^1 function

$$V(x)=\bar{V}(z)=\frac{1}{2}\sum_{i=1}^{N}z_i^TP_iz_i \quad (23)$$

The total time derivative of V along any trajectory of the uncertain large-scale system (1) under (16) is then given by

$$\begin{aligned}L(z,\ t)&\equiv\dot{V}(x)=\dot{\bar{V}}(z)\\&=-\frac{1}{2}\sum_{i=1}^{N}z_i^TQ_iz_i+\sum_{i=1}^{N}[-B_i^TP_iz_i(1+\bar{l}_i)p_i+B_i^TP_iz_i(\bar{l}_iK_iz_i+\omega_{i1})]+\sum_{i=1}^{N}[-(1+\bar{l}_i)\gamma_i(B_i^TP_iz_i)^TB_i^TP_iz_i+B_i^TP_iz_i\omega_{i2}]\end{aligned} \quad (24)$$

With regard to the second term of Eq. (24), if $|\ \mu_i\ |>\varepsilon_i$,

$$-B_i^TP_iz_i(1+\bar{l}_i)p_i+B_i^TP_iz_i(\bar{l}_iK_iz_i+\omega_{i1})$$

$$\leq -(1+\rho_{li})|\mu_i| + (1+\rho_{li})|\mu_i| = 0 \qquad (25)$$

and if $|\mu_i| \leq \varepsilon_i$,

$$\begin{aligned} & -B_i^T P_i z_i (1+\bar{l}_i) p_i \\ & + B_i^T P_i z_i (\bar{l}_i K_i z_i + \omega_{i1}) \\ & \leq -\frac{(1+\rho_{li})}{\varepsilon_i} |\mu_i|^2 + (1+\rho_{li}) | |\mu_i| \\ & \leq \frac{(1+\rho_{li})\varepsilon_i}{4} \end{aligned} \qquad (26)$$

Consequently, for all $z \in \mathbf{R}^n$, the second term is bounded by $\sum_{i=1}^{N}(1+\rho_{li})\varepsilon_i/4$. Now, we perform the worst case analysis of the third term in Eq. (24):

$$\begin{aligned} & \sum_{i=1}^{N}[-(1+\bar{l}_i)\gamma_i (B_i^T P_i z_i)^T B_i^T P_i z_i \\ & + B_i^T P_i z_i \omega_{i2}] \\ & \leq \sum_{i=1}^{N}[-(1+\rho_{li})\gamma_i |B_i^T P_i z_i|^2 \\ & + |B_i^T P_i Z_i| (\sum_{j=1}^{N} a_{ij}\|z_j\| + b_i)] \end{aligned} \qquad (27)$$

Using the algebraic inequality

$$\alpha\beta \leq \frac{1}{2}\alpha^2 + \frac{1}{2}\beta^2, \quad \alpha, \beta \in \mathbf{R}_+ \qquad (28)$$

one has

$$\begin{aligned} & \sum_{i=1}^{N} |B_i^T P_i z_i| (\sum_{j=1}^{N} a_{ij}\|z_j\|) \\ & = \sum_{i=1}^{N}\sum_{j=1}^{N} |B_i^T P_i z_i| \delta_j^{\frac{1}{2}} a_{ij}^{\frac{1}{2}} \delta_j^{-\frac{1}{2}} a_{ij}^{\frac{1}{2}} \|z_j\| \\ & \leq \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{1}{2}(\delta_j a_{ij} |B_i^T P_i z_i|^2 \\ & + \delta_j^{-1} a_{ij} \|z_j\|^2) \end{aligned} \qquad (29)$$

where δ_i is a positive constant. Therefore, the total time derivative of V satisfies the following inequality

$$\begin{aligned} L(z, t) \leq & -\frac{1}{2}\sum_{i=1}^{N}\Big[\lambda_{\min}(Q_i) \\ & -\sum_{j=1}^{N}\delta_i^{-1}a_{ji}\Big]\|z_i\|^2 \\ & +\sum_{i=1}^{N}\frac{(1+\rho_{li})\varepsilon_i}{4} \\ & +\sum_{i=1}^{N}\Big[\Big(-(1+\rho_{li})\gamma_i \\ & +\frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}\Big)|B_i^T P_i z_i|^2 \\ & + b_i |B_i^T P_i z_i|\Big] \end{aligned} \qquad (30)$$

Let

$$\eta_i \equiv (1+\rho_{li})\gamma_i - \frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}$$

and

$$\xi_i \equiv \frac{1}{2}\Big[\lambda_{\min}(Q_i) - \sum_{j=1}^{N}\delta_i^{-1}a_{ji}\Big]$$

Since δ_i and γ_i are chosen to satisfy the inequalities (18) and (19), ξ_i and η_i must be positive constants. Then the last term in Eq. (30) is bounded by

$$\sum_{i=1}^{N}\frac{b_i^2}{4\eta_i} = \sum_{i=1}^{N}\frac{b_i^2}{4(1+\rho_{li})\gamma_i - 2\sum_{j=1}^{N}\delta_j a_{ij}}$$

Consequently, one has

$$\begin{aligned} L(z, t) & \leq -\sum_{i=1}^{N}\xi_i\|z_i\|^2 + \kappa \\ & \leq -\xi\|z\|^2 + \kappa \end{aligned} \qquad (31)$$

where

$$\xi \equiv \min_{i\in N}\{\xi_i\}$$

$$\kappa \equiv \sum_{i=1}^{N}\Big[(1+\rho_{li})\frac{\varepsilon_i}{4} + \frac{b_i^2}{4\eta_i}\Big]$$

Now, one has

$$L(z, t) < 0 \qquad (32)$$

for all $(z, t) \in \mathbf{R}^n \times \mathbf{R}$ such that

$$\|z\| > \sqrt{\frac{\kappa}{\xi}} \equiv \bar{s}_1 \qquad (33)$$

Let

$$\begin{aligned} T^{-1}(z) & \equiv [T_1^{-T}(z_1), \ T_2^{-T}(z_2), \\ & \cdots, T_N^{-T}(z_N)]^T \\ s_1 & \equiv \sup_{\|E\|=\bar{s}_1}\|T^{-1}(z)\| \end{aligned} \qquad (34)$$

Finally, one has

$$\dot{V}(x) < 0 \qquad (35)$$

for all $x \in \mathbf{R}^n$ such that

$$\|x\| > s_1 \qquad (36)$$

Theorem 1: Subject Assumption 1 and 2, the uncertain large-scale system under the proposed decentralized robust controls Eq. (16) is practically stable.

Proof : The right-hand side of Eqs. (1) under (16) is Caratheodory by using some standard results in analysis. The Lyapunov-based argument for the feedback linearized system is already shown above. The result follows based on the standard arguments in Corless and Leitmann

(1981).

5. Adaptive Robust Control Design

If the bound of uncertainty is unknown, the aforementioned robust control is not applicable. Then, it is desirable to introduce an adaptive scheme which is capable of tracking the bound. Additional assumptions will be imposed on the uncertainty.

Assumption 3 : There exist (unknown) constants d_i and e_i such that

$$\max_{\sigma_i \in \Sigma_i} | \omega_{i1}(z_i, \sigma_i) | \leq d_i \|z_i\| + e_i \quad (37)$$

for all $(z_i, \sigma_i) \in R^{n_i} \times \Sigma_i$.

Assumption 4: Each subsystem does not have uncertainty in the input matrix, that is,

$$\Delta g_i(x_i, \sigma_i) = 0 \quad (38)$$

Subject to Assumption 4, the uncertain system (1) can be rewritten in a compact form as

$$\begin{aligned} S: \dot{x} &= f(x) + \Delta f(x, \sigma) \\ &\quad + G(x)u + R(x, \sigma) \\ x(t_0) &= x_0 \end{aligned} \quad (39)$$

The feedback linearized system (11) also can be rewritten as following

$$\begin{aligned} S_i: \dot{z}_i &= A_i z_i + B_i \nu_i + B_i \omega_{i1}(z_i, \sigma_i) \\ &\quad + B_i \omega_{i2}(z, \sigma_i) \\ z_i(t_0) &= z_{i0} \end{aligned} \quad (40)$$

The following class of adaptive robust controls is proposed

$$\nu_i(z_i, \hat{\gamma}_i) = K_i z_i + \hat{p}_i(z_i, \hat{\gamma}_i) \quad (41)$$

where

$$\hat{p}_i(z_i, \hat{\gamma}_i) = -\hat{\gamma}_i B_i^T P_i z_i \quad (42)$$

The adaptive parameter $\hat{\gamma}_i$ is governed by the following dynamics

$$\begin{aligned} \dot{\hat{\gamma}}_i(t) &= l_{i1} | B_i^T P_i z_i(t) |^2 - l_{i2} \hat{\gamma}_i(t) \\ \hat{\gamma}_i(t_0) &= \hat{\gamma}_{i0} \end{aligned} \quad (43)$$

where l_{i1} and l_{i2} are arbitrary positive constants.

Remark : The adaptive scheme (43) is a leakage-type adaptive scheme which belongs to the σ-modification class (Chen, 1990 ; Ioannou and Kokotovic, 1983).

For convenience, let

$$\hat{\gamma} \equiv [\hat{\gamma}_1, \hat{\gamma}_2, \cdots, \hat{\gamma}_N]^T \in R^N$$

$$\hat{\gamma}(t_0) = \hat{\gamma}_0 \quad (44)$$

$$\gamma \equiv [\gamma_1, \gamma_2, \cdots, \gamma_N]^T \in R^N \quad (45)$$

Theorem 2: Suppose that the uncertain large-scale system (39) satisfies Assumption 1-4 but with a_{ij} and b_i in Eq. (15) unknown. If the system (39) is subject to the decentralized adaptive robust control (41)~(43), then the overall combined controlled system (39) and (43) is practically stable.

Proof: The Lyapunov function candidate is taken as

$$\begin{aligned} V(x, \hat{\gamma}) &= \bar{V}(z, \hat{\gamma}) \\ &= \frac{1}{2}\sum_{i=1}^{N} z_i^T P_i z_i + \sum_{i=1}^{N} k_i (\hat{\gamma}_i - \gamma_i)^2 \end{aligned} \quad (46)$$

where k_i is a positive constant. The choice of values of k_i and γ_i will be stated later. The total time derivative of $V(x, \hat{\gamma})$ along any trajectory of the combined system is given by:

$$\begin{aligned} L(z, \hat{\gamma}, t) &\equiv \dot{V}(x, \hat{\gamma}) = \dot{\bar{V}}(z, \hat{\gamma}) \\ &= \sum_{i=1}^{N} \dot{z}_i^T P_i z_i + \sum_{i=1}^{N} 2k_i (\hat{\gamma}_i - \gamma_i) \\ &\quad \dot{\hat{\gamma}}_i \\ &= -\frac{1}{2}\sum_{i=1}^{N} z_i^T Q_i z_i + \sum_{i=1}^{N} (-\gamma_i \\ &\quad | B_i^T P_i z_i |^2 + B_i^T P_i z_i \omega_{i1}) \\ &\quad - \sum_{i=1}^{N} (\hat{\gamma}_i - \gamma_i) \, | B_i^T P_i z_i |^2 \\ &\quad + \sum_{i=1}^{N} B_i^T P_i z_i \omega_{i2} \\ &\quad + \sum_{i=1}^{N} [2k_i l_{i1} (\hat{\gamma}_i - \gamma_i) \, | B_i^T P_i z_i |^2 \\ &\quad - 2k_i l_{i2} (\hat{\gamma}_i - \gamma_i) \hat{\gamma}_i] \end{aligned} \quad (47)$$

The constant k_i is chosen such that

$$2k_i l_{i1} = 1 \quad (48)$$

Therefore, using Eqs. (15), (28), and (37), one has

$$\begin{aligned} L(z, \hat{\gamma}, t) &\leq -\frac{1}{2}\sum_{i=1}^{N}\left[\lambda_{\min}(Q_i) - \sum_{j=1}^{N} \delta_i^{-1} a_{ji}\right] \|z_i\|^2 \\ &\quad + \sum_{i=1}^{N}\left[-\left(\gamma_i - \frac{1}{2}\sum_{j=1}^{N} \delta_j a_{ij}\right) | B_i^T P_i z_i |^2 \right. \\ &\quad \left. + (d_i \|z_i\| + e_i + b_i) \, | b_i^T P_i z_i | \right] \\ &\quad - \sum_{i=1}^{N} 2k_i l_{i2} (\hat{\gamma}_i - \gamma_i)^2 \\ &\quad - \sum_{i=1}^{N} 2k_i l_{i2} (\hat{\gamma}_i - \gamma_i) \gamma_i \end{aligned} \quad (49)$$

First, The constant δ_i is chosen such that

$$\frac{1}{2}\left[\lambda_{\min}(Q_i)-\sum_{j=1}^{N}\delta_i^{-1}a_{ji}\right]>\nu_i \tag{50}$$

where ν_i is some positive constant. Next, the constant γ_i in Eq. (46) is chosen such that

$$\gamma_i>\frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}+\frac{d_i^2}{4\nu_i} \tag{51}$$

Therefore, with these choices,

$$\gamma_i-\frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}>0 \tag{52}$$

$$\alpha_i\equiv\frac{1}{2}\lambda_{\min}(Q_i)-\frac{1}{2}\sum_{j=1}^{N}\delta_i^{-1}a_{ji}-\frac{d_i^2}{4\left(\gamma_i-\frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}\right)}>0 \tag{53}$$

Subject to Eq. (52), the maximum value of the second term in Eq. (49) is given by

$$\frac{(d_i\|z_i\|+e_i+b_i)^2}{4\left(\gamma_i-\frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}\right)}$$

This leads to

$$\begin{aligned}L(z,\ \hat{\gamma},\ t)\leq&-\sum_{i=1}^{N}(\alpha_i\|z_i\|^2-\beta_i\|z_i\|-\psi_i)\\&-\sum_{i=1}^{N}2k_i l_{i2}(\hat{\gamma}_i-\gamma_i)^2\\&+\sum_{i=1}^{N}2k_i l_{i2}|\gamma_i|\cdot|\hat{\gamma}_i-\gamma_i|\\=&-\sum_{i=1}^{N}(\alpha_i\|z_i\|^2+2k_i l_{i2}(\hat{\gamma}_i-\gamma_i)^2)\\&+[\beta_1\cdots\beta_N 2k_1 l_{12}|\gamma_i|\cdots\\&2k_N l_{N2}|\gamma_N|]\\&\cdot[\|z_1\|\cdots\|z_N\|\,|\hat{\gamma}_1-\gamma_1|\cdots|\hat{\gamma}_N\\&-\gamma_N|]^T+\sum_{i=1}^{N}\psi_i\end{aligned} \tag{54}$$

where

$$\beta_i\equiv\frac{2d_i(e_i+b_i)}{4\left(\gamma_i-\frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}\right)} \tag{55}$$

$$\psi_i\equiv\frac{(e_i+b_i)^2}{4\left(\gamma_i-\frac{1}{2}\sum_{j=1}^{N}\delta_j a_{ij}\right)} \tag{56}$$

Let

$$\begin{aligned}&\xi\equiv[z^T,\ \hat{\gamma}_1-\gamma_1,\cdots,\ \hat{\gamma}_N-\gamma_N]^T\\&m_1\equiv\min_{i\in N}\{\alpha_i,\ 2k_i l_{i2}\}\\&m_2\equiv[\beta_1^2+\cdots+\beta_N^2+(2k_1 l_{12})^2+\cdots\\&\quad+(2k_N l_{N2})^2]^{\frac{1}{2}}\\&m_3\equiv\sum_{i=1}^{N}\psi_i\end{aligned}$$

Combining the results of Eqs. (53)~(56), one has

$$\bar{L}(\xi,\ t)\equiv L(z,\ \hat{\gamma},\ t)\leq-m_1\|\xi\|^2+m_2\|\xi\|+m_3 \tag{57}$$

Now, it is shown that

$$\bar{L}(\xi,\ t)<0 \tag{58}$$

for all $(\xi,\ t)\in\mathbf{R}^{n\times N}\times\mathbf{R}$ such that

$$\|\xi\|>\frac{m_2+\sqrt{m_2^2+4m_1m_3}}{2m_1}\equiv\bar{s}_2 \tag{59}$$

Let

$$\begin{aligned}s_2\equiv\sup\nolimits_{\|\xi\|=\bar{s}_2}\|[&T^{-T}(z),\ \hat{\gamma}_1-\gamma_1,\\&\hat{\gamma}_2-\gamma_2,\cdots,\ \hat{\gamma}_N-\gamma_N]\|\end{aligned} \tag{60}$$

Finally, one has

$$\dot{V}(x,\ \hat{\gamma})<0 \tag{61}$$

for all $(x,\ \hat{\gamma})\in\mathbf{R}^n\times\mathbf{R}^N$ such that

$$\begin{aligned}\|[x^T,\ &\hat{\gamma}_1-\gamma_1,\ \hat{\gamma}_2-\gamma_2,\cdots,\\&\hat{\gamma}_N-\gamma_N]\|>s_2\end{aligned} \tag{62}$$

The four properties of Definition 1 then follow (Leitmann, 1981 ; Chen, 1990).

Remark : The constants δ_i and γ_i are chosen such that Eqs. (52) and (53) are satisfied. But these constants do not appear in the control, (42) and (43), and show only in the proof of Theorem 2. Therefore, only their existence needs to be proven.

6. Example

Consider the following system:

$$\begin{aligned}S_1:\ &\dot{x}_{11}=x_{11}^2+x_{12}\\&\dot{x}_{12}=-c_1(t)x_{11}+u_1+c_2(t)x_{21}\end{aligned} \tag{63}$$

$$\begin{aligned}S_2:\ &\dot{x}_{21}=x_{22}\\&\dot{x}_{22}=-x_{21}^3+u_2+c_3(t)x_{11}\end{aligned} \tag{64}$$

where $0.5\leq c_i(t)\leq 1.5$, $i=1,\ 2,\ 3$. The nominal values of the time-varying uncertain parameters $c_i(t)$'s are taken as 1.

With the following state transformation and the control inputs

$$\begin{aligned}&z_{11}=x_{11},\quad z_{12}=x_{11}^2+x_{12},\quad z_{21}=x_{21},\\&z_{22}=x_{22},\end{aligned} \tag{65}$$

$$\begin{aligned}&u_1=-2x_{11}^3-2x_{11}x_{12}+x_{11}+\nu_1,\\&u_2=x_{21}^3+\nu_2,\end{aligned} \tag{66}$$

we can get

$$\dot{z}_{11}=z_{12}$$
$$\dot{z}_{12}=\nu_1+(1-c_1)z_{11}+c_2z_{21}$$
$$\dot{z}_{21}=z_{22}$$
$$\dot{z}_{22}=\nu_2+c_3z_{11} \tag{67}$$

The uncertain portions are bounded as following:

$$|\omega_{11}|=|(1-c_1)z_{11}|\le 0.5\|z_1\|$$
$$|\omega_{12}|=|c_2z_{21}|\le 1.5\|z_2\|$$
$$|\omega_{21}|=0$$
$$|\omega_{22}|=|c_3z_{11}|\le 1.5\|z_1\| \tag{68}$$

Since Eqs. (15) and (37) are satisfied, both non-adaptive and adaptive robust controls are applicable. If we choose

$$K_i=[-1\ \ -2],\ Q_i=-I,\ i=1,\ 2,$$

then,

$$P_i=\begin{bmatrix}1.5 & 0.5\\ 0.5 & 0.5\end{bmatrix},\ i=1,\ 2.$$

The nonlinear gain $\rho_i(z_i)$ in Eq. (21) is now

$$\rho_1(z_1)=0.5\|z_1\|,$$
$$\rho_2(z_2)=0. \tag{69}$$

The non-adaptive robust controllers are

$$u_1=-2x_{11}^3-2x_{11}x_{12}+x_{11}-1.6z_{11}$$
$$-2.6z_{12}-p_1(z_1),$$
$$u_2=x_{21}^3-1.6z_{21}-2.6z_{22}-p_2(z_2). \tag{70}$$

The adaptive robust controllers are

$$u_1=-2x_{11}^3-2x_{11}x_{12}+x_{11}-z_{11}-2z_{12}$$
$$-0.5\hat{\gamma}_1(z_{11}+z_{12}),$$
$$u_2=x_{21}^3-z_{21}-2z_{22}-0.5\hat{\gamma}_2(z_{21}+z_{22}), \tag{71}$$

with the following dynamics of the adaptive parameters $\hat{\gamma}_i$'s:

$$\dot{\hat{\gamma}}_1=l_{11}(0.5z_{11}+0.5z_{12})^2-l_{12}\hat{\gamma}_1,$$
$$\dot{\hat{\gamma}}_2=l_{21}(0.5z_{21}+0.5z_{22})^2-l_{22}\hat{\gamma}_2. \tag{72}$$

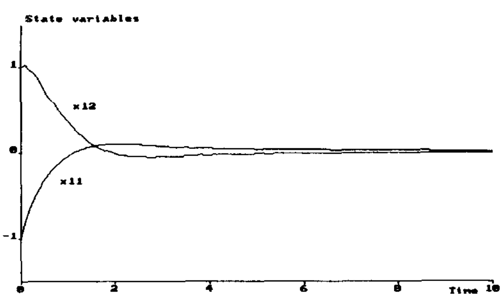

Fig. 1 Response of subsystem 1 under non-adaptive robust control

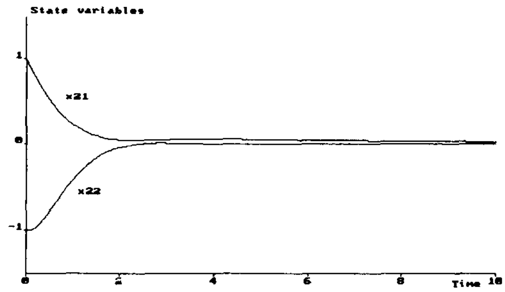

Fig. 2 Response of subsystem 2 under non-adaptive robust control

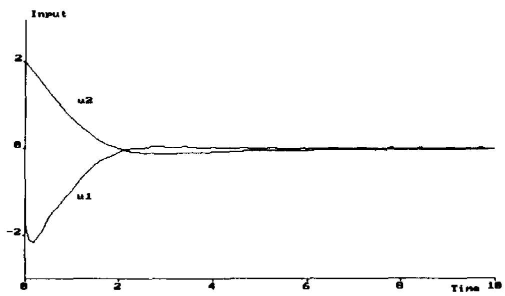

Fig. 3 Control history of non-adaptive robust control

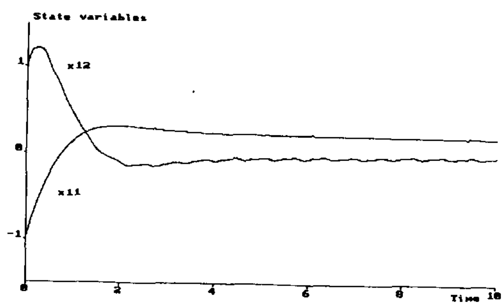

Fig. 4 Response of subsystem 1 under adaptive robust control

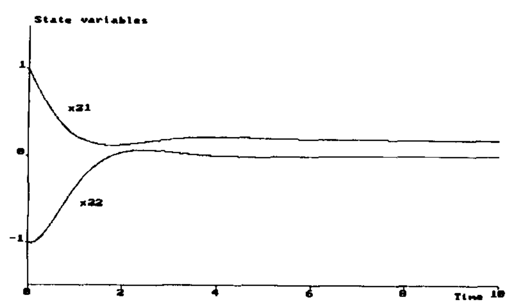

Fig. 5 Response of subsystem 2 under adaptive robust control

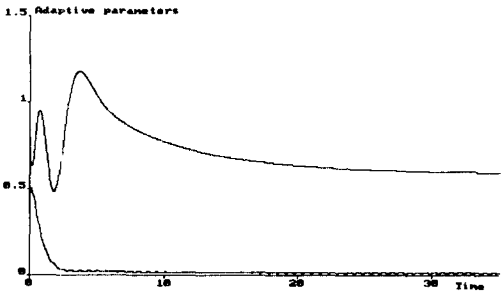

Fig. 6 Histories of adaptive parameters

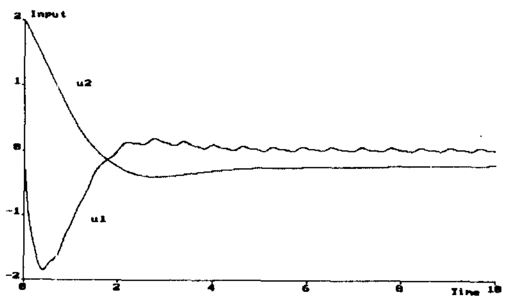

Fig. 7 Control history of adaptive robust control

Simulation results are shown graphically in Figs. 1~7. Figures 1~3 represent the system responses and inputs for the non-adaptive robust controllers in Eq. (70). Figures 4~7 represent the system responses, adaptive parameters, and inputs for the adaptive robust controllers in Eq. (71).

7. Conclusions

It has been demonstrated that feedback linearization is a systematic controller design method for uncertain nonlinear large-scale systems. The uncertainties which the systems possess, can be (fast) time-varying. No statistical information of the uncertainties is assumed. If the robust control is applied alone, some difficulties may arise on how to choose a suitable Lyapunov function for the nonlinear nominal system. This is the most frequently noticeable when the dimension of the system is high. Feedback linearization technique is used with robust control in order to find the Lyapunov function of the nominal system systematically. Two classes of decentralized robust control, namely, non-adaptive and adaptive, have been constructed. In non-adaptive control, all the bounds of uncertainties are given and the control gain parameters are dependent on the bounds. The adaptive control scheme is applicable to the system which has no uncertainty in the input matrix. However, this scheme does not need to know the bound of uncertainty.

References

Barmish, B. R., Corless, M. and Leitmann, G., 1983, "A New Class of Stabilizing Controllers for Uncertain Dynamical Systems," *SIAM Journal of Control and Optimization*, Vol. 21, No. 2, pp. 246~255.

Calvet, J. P. and Arkun, Y., 1989, "Robust Control Design for Uncertain Nonlinear Systems under Feedback Linearization," *Proc. 28th IEEE Conf. on Decision and Control*, Tampa, FL, pp. 102~106.

Chen, Y. H., 1988, "Decentralized Robust Control System Design for Large-scale Uncertain Systems," *International Journal of Control*, Vol. 47, No. 5, pp. 1195~1205.

Chen, Y. H., 1990, "Robust Control System Design: Non-Adaptive Versus Adaptive," *International Journal of Control*, Vol. 51, No. 6, pp. 1457~1477.

Chen, Y. H. and Leitmann, G., 1987, "Robustness of Uncertain Systems in the Absence of Matching Assumptions," *International Journal of Control*, Vol. 45, No. 5, pp. 1527~1542.

Chen, Y. H. and Han, M. C., 1993, "Decentralized Control Design for Interconnected Uncertain Systems," *Control and Dynamic Systems*, Vol. 56, Academic Press, pp. 219~266.

Corless, M. and Leitmann, G., 1981, "Continuous State Feedback Guaranteeing Uniform Ultimate Boundedness for Uncertain Dynamic Systems," *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 5, pp. 1139~1144.

Corless, M. and Leitmann, G., 1983, "Adaptive Control of Systems Containing Uncertain Functions and Unknown Functions with Uncertain Bound," *Journal of Optimization Theory and Applications*, Vol. 41, pp. 155~168.

Gavel, D. T. and Siljak, D. D., 1989,

"Decentralized Adaptive Control : Structural Conditions for Stability," *IEEE Transactions on Automatic Control*, Vol. 129, No. 4, pp. 413~426.

Han, M. C. and Chen, Y. H., 1991, "Decentralized Robust Control of Nonlinear Systems with Bounded Time-Varying Uncertainties," *Control Theory and Advanced Technology*, Vol. 7, No. 4, pp. 609~628.

Han, M. C. and Chen, Y. H., 1992, "Polynominal Robust Control Design for Uncertain Systems," *Automatica*, Vol. 28, No. 4, pp. 809~814.

Han, M. C. and Chen, Y. H., 1992, "Decentralized Robust Control Design with Strong Interconnections," *Proc. American Control Conference*, pp. 3279~3283.

Ikeda, M. and Siljak, D. D., 1990, "Robustness and Optimality of Linear-Quadratic Control for Nonlinear Systems," *Proc. 11th IFAC World Congress*, Tallinn.

Ioannou, P. A. and Kokotovic, P. V., 1983, *Adaptive Systems with Reduced Models*, Spriger-Verlag, Berlin.

Leitmann, G., 1981, "On the Efficacy of Nonlinear Control in Uncertain Linear Systems," *Journal of Dynamic Systems, Measurement, and Control*, Vol. 103, pp. 95~102.

Ohta, Y., Siljak, D. D. and Matsumoto, T., 1986, "Decentralized Control Using Quasi-Block Diagonal Dominance of Transfer Function Matrices," *IEEE Transactions on Automatic Control*, Vol. 31, No. 5, pp. 420~430.

Park, C. G. and Lee, J. G., 1993, "Stability of an Overlapping Decentralized Controller," *Proc. 1993 Korean Automatic Control Conference*, pp. 943~946.

Sastry, S. S. and Isidori, A., 1989, "Adaptive Control of Linearizable Systems," *IEEE Transactions on Automatic Control*, Vol. 34, pp. 1123~1131.

Siljak, D. D., 1989, "Parameter Space Methods for Robust Control Design: a Guided Tour," *IEEE Transactions on Automatic Control*, Vol. 34, No. 7, pp. 674~688.

Sira-Ramirez, H., 1986, "Variable Structure Control of Nonlinear Systems Through Simplified Uncertain Models," *Proc. 25th IEEE Conf. on Decision and Control*, Athens, Greece, pp. 2037~2041.

Spong, M. W. and Sira-Ramirez, H., 1986, "Rbust Control Design Techniques for a Class of Nonlinear Systems," *Proc. American Control Conference*, Seattle, WA, pp. 1515~1522.

Taylor, D. G., Kokotovic, P. V., Marino, R. and Kanellakopoulos, I., 1989, "Adaptive Regulation of Nonlinear System With Unmodeled Dynamics," *IEEE Transactions on Automatic Control*, Vol. 34, pp. 405~412.